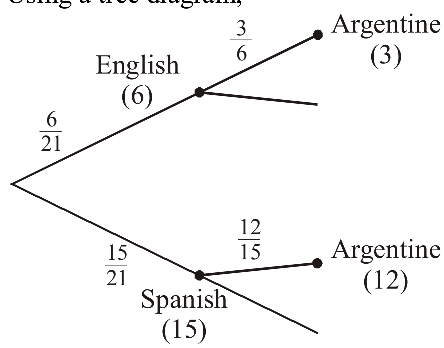
CONDITIONAL PROBABILITY – BINOMIAL DISTRIBUTION MARKSCHEME

1. Using a tree diagram,



(M2)

Let $p(S)$ be the probability that the pupil speaks Spanish.
Let $p(A)$ be the probability that the pupil is Argentine.

Then, from diagram,

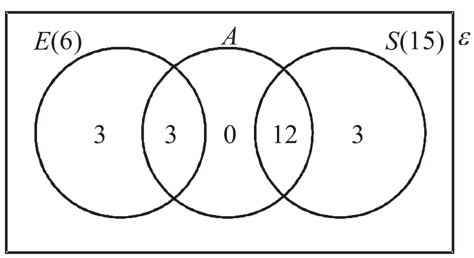
$p(S|A) = \frac{12}{15}$ (A1)

$= \frac{4}{5}$ (A1)

OR $p(S|A) = \frac{p(S \cap A)}{p(A)}$ (M1)

$= \frac{12}{21} \Big/ \frac{15}{21}$ (M1)(A1)

OR



$p(S|A) = \frac{12}{15}$ (A1)

$= \frac{4}{5}$ (A1)

[4]

2. Let D be the event that the patient has the disease and S be the event that the new blood test shows that the patient has the disease. Let D' be the complement of D, *i.e.* the patient does not have the disease.
Now the given probabilities can be written as
$p(S|D) = 0.99$, $p(D) = 0.0001$, $p(S|D') = 0.05$. (A1)(A1)(A1)

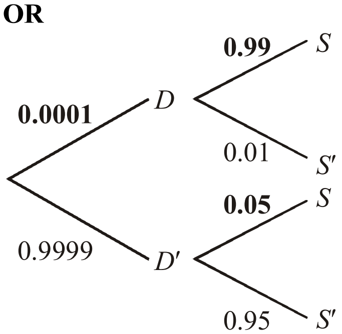
Since the blood test shows that the patient has the disease, we are required to find $p(D \mid S)$.
By Bayes' theorem,

$$p(D \mid S) = \frac{p(S|D)\,p(D)}{p(S|D)\,p(D) + p(S|D')\,p(D)} \qquad \text{(M1)}$$

$$= \frac{(0.99)(0.0001)}{(0.99)(0.0001) + (0.05)(1 - 0.0001)} \qquad \text{(M1)}$$

= 0.001976...= 0.00198 (3 s.f.) (A1)

OR



(A3)

Note: *Award (A1) for 0.99, (A1) for 0.0001, (A1) for 0.05*

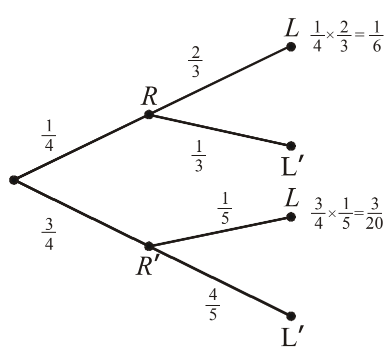
Therefore $p(S) = 0.0001 \times 0.99 + 0.9999 \times 0.05$
$= 0.0500939$ (A1)

$$p(D \mid S) = \frac{0.0001 \times 0.99}{0.0500939} \qquad \text{(M1)}$$

$= 0.00198$ (3 s.f.) (A1)

[6]

3. Let $P(R \mid L)$ be the probability that it is raining given that the girl is late.



$$P(R \mid L) = \frac{P(R \cap L)}{P(L)}$$

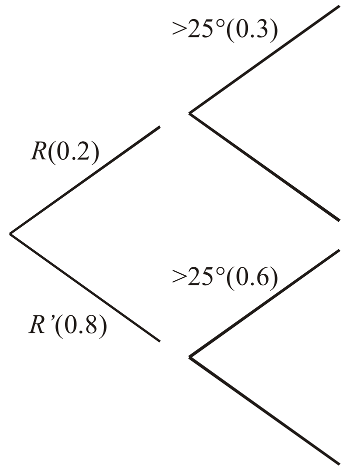
$$P(R \mid L) = \frac{1/6}{1/6 + 3/20} \qquad \text{(M1)(A1)}$$

(using a tree diagram or by calculation)

$$= \frac{10}{19} \qquad \text{(A1)}$$

[3]

4.



(M2)

P (> 25°) = 0.2 × 0.3 + 0.8 × 0.6 = 0.54 (M1)(A1)

$P(R \mid >25°) = \frac{0.06}{0.54} = \frac{1}{9}$ (or 0.111) (M1)(A1) (C6)

[6]

5. (a) Probability = 0.2 × 0.66 + 0.8 × 0.75 (M1)(A1)

= 0.732 (A1) (C3)

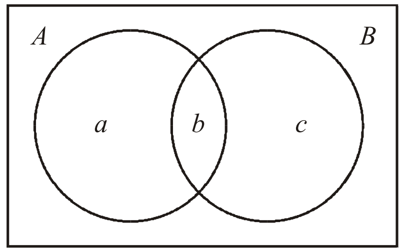
(b) Probability = $\frac{\text{P(Mon} \cap \text{catches train)}}{\text{P(catches train)}}$ (M1)

$= \frac{0.2 \times 0.66}{0.732}$ (A1)

$= 0.180\left(= \frac{11}{61}\right)$ (A1) (C3)

[6]

6. **METHOD 1**



$\frac{c}{b+c}=\frac{1}{3} \Rightarrow c=\frac{b}{2}$ (A1)

$a+b+c=1 \Rightarrow a+\frac{3}{2}b=1$ (A1)

also $a+b=\frac{6}{7}$

$\Rightarrow b=\frac{2}{7} \quad c=\frac{1}{7} \quad (a=\frac{4}{7}$ not needed) (A1)(A1)

$\mathrm{P}(B)=b+c=\frac{3}{7} \quad (=0.429)$ (M1)(A1) (C6)

METHOD 2

$\mathrm{P}(A' | B)=\frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (M1)

$\frac{1}{3}\mathrm{P}(B)=\mathrm{P}(A' \cap B)$ (A1)

$\mathrm{P}(A' \cap B)+\mathrm{P}(A)=1$ (M1)(A1)

$\frac{1}{3}\mathrm{P}(B)+\frac{6}{7}=1$ (A1)

$\mathrm{P}(B)=\frac{3}{7} \quad (=0.429)$ (A1) (C6)

[6]

7. (a) P(all ten cells fail) = 0.8^{10} = 0.107. (M1)(A1) 2

(b) P(satellite is still operating at the end of one year)
= 1 – P (all ten cells fail within one year) (M1)
= 1 – 0.107
= 0.893. (A1) 2

(c) P(satellite is still operating at the end of one year)
$= 1 - 0.8^n$. (C1)

We require the smallest n for which $1 - 0.8^n \geq 0.95$. (M1)

Thus, $0.8^n \leq 0.05$

$\left(\frac{5}{4}\right)^n \geq 20$

$n \geq \frac{\log 20}{\log 1.25} = 13.4$ (M1)(A1)

Therefore, 14 solar cells are needed. (C1) 5

[9]

8. $n = 1800$, $p = \frac{2}{3}$

(a) $E(X) = np = 1200$ (A1) (C1)

(b) $SD(X) = \sqrt{np(1-p)} = \sqrt{1200 \times \frac{1}{3}} = 20$ (M1)(A1) (C2)

[3]

9. Let p be the probability of choosing a student who travels to school by bus.
Let X be the random variable "the number of students who travel to school by bus." (M1)

Then $X \sim B(n, p)$ with $n = 5$ and $p = \frac{1}{3}$

Therefore $P(X = 3) = \binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$ (using formulae and statistical tables) (A1)

$= \frac{40}{243}$ or 0.165 (A1)

[3]

= 20 (M1)(A1) (C2)

[3]

10. (a) Probability $= \binom{6}{4} \times (0.4)^4 \times (0.6)^2$ (M1)(A1)

$= 0.138 \left(\text{accept } \frac{432}{3125} \text{ or } 0.13824\right)$ (A2) (C4)

(b) Probability $= (0.6)^2 \times 0.4 = 0.144 \left(\text{or } \frac{18}{125}\right)$ (M1)(A1) (C2)

[6]

11. METHOD 1

X is Binomial

$n = 5$ $p = 0.4$ (A1)(A1)

$P(X \leq 3) = 1 - P(X = 4) - P(X = 5)$ (M1)

$= 1 - 0.0768 - 0.01024$ (A1)(A1)

$= 0.91296...$ (0.913 to 3 s.f.) (A1) (C6)

METHOD 2

$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$ (M1)

$= 0.07776 + 0.2592 + 0.3456 + 0.2304$ (A2)

$= 0.91296...$ (0.913 to 3 s.f.) (A1) (C6)

[6]