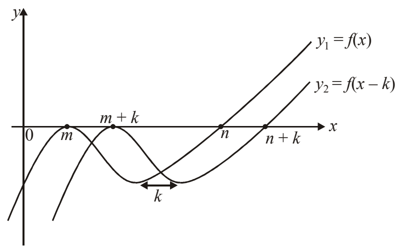
IB HL Graphing Problems MARKSCHEME

1.



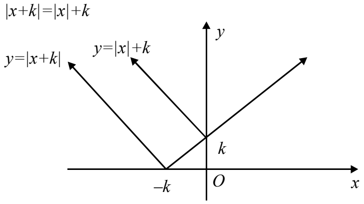
(A2)(A2) (C4)

Notes: *The graph of y_2 is y_1 shifted k units to the right.*
Award (A2) for the correct graph.
Award (A1) for indicating each point of intersection with the x-axis ie $(m + k, 0)$ and $(n + k, 0)$.
Award (C4) if the graph of y_2 is drawn correctly and correctly labelled with $m + k$ and $n + k$.

[4]

2.

$|x+k|=|x|+k$



(M2)

From the graph, $x \geq 0$. (A2)

OR $|x + k| = |x| + k$

$\Rightarrow |x + k|^2 = (|x| + k)^2$ (M1)

$\Rightarrow x^2 + 2kx + k^2 = x^2 + 2k|x| + k^2$ (M1)

$\Rightarrow x = |x|$ (M1)

$\Rightarrow x \geq 0$ (A1) (C4)

[4]

3. (a) Let $g(x) = ax^3 + bx^2 + cx + d$

$g(0) = -4 \Rightarrow d = -4$ (A1)

$g'(x) = 3ax^2 + 2bx + c$ (M1)

$g'(0) = 0 \Rightarrow c = 0$ (A1)

$g(-2) = 0 \Rightarrow -8a + 4b = 4$

$g'(-2) = 0 \Rightarrow 12a - 4b = 0$ (M1)

$4a = 4$

$a = 1$ (A1)

$b = 3$ (A1)

Therefore, $g(x) = x^3 + 3x^2 - 4$ (AG) 6

(b) Under reflection in the y-axis, the graph of $y = -x^3 + 3x^2$ is mapped onto the graph of

$y = -(-x)^3 + 3(-x)^2$ (M1)

ie $y = x^3 + 3x^2$. (A1)

Under translation $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$, the graph of $y = x^3 + 3x^2$ is mapped onto the graph of

$y = h(x) = (x + 1)^3 + 3(x + 1)^2 - 1$ (M1)

$= x^3 + 3x^2 + 3x + 1 + 3x^2 + 6x + 3 - 1$ (A1)

$h(x) = x^3 + 6x^2 + 9x + 3$ (A1) 5

(c) The graph of $y = -x^3 + 3x^2$ is mapped onto the graph of $y = x^3 + 3x^2 - 4$, with point A mapped onto point A', using the following combination of transformations:

Reflection in the x-axis (A1)

followed by the translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$. (A2) 3

(or vice versa.)

[14]

4. (a) (i) $y = \dfrac{a + b\sin x}{b + a\sin x}$, $0 < a < b$

$$\frac{dy}{dx} = \frac{(b + a\sin x)(b\cos x) - (a + b\sin x)(a\cos x)}{(b + a\sin x)^2}$$ (M1)(C1)

$$= \frac{b^2\cos x + ab\sin x\cos x - a^2\cos x - ab\sin x\cos x}{(b + a\sin x)^2}$$ (M1)(C1)

$$= \frac{(b^2 - a^2)\cos x}{(b + a\sin x)^2}$$ (AG) 4

(ii) $\dfrac{dy}{dx} = 0 \Rightarrow \cos x = 0$ since $b^2 - a^2 \neq 0$.

This gives $x = \dfrac{\pi}{2}$ $(+\pi k, k \in \mathbb{Z})$ (M1)(C1)

When $x = \dfrac{\pi}{2}$, $y = \dfrac{a + b}{b + a} = 1$,

and when $x = \dfrac{3\pi}{2}$, $y = \dfrac{a - b}{b - a} = -1$.

Therefore, maximum $y = 1$ and minimum $y = -1$. (A2) 4

(iii) A vertical asymptote at the point x exists if and only if $b + a\sin x = 0$. (R1)

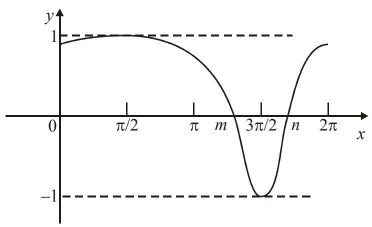
Then, since $0 < a < b$, $\sin x = -\dfrac{b}{a} < -1$, which is impossible. (R1)

Therefore, no vertical asymptote exists. (AG) 2

(b) (i) y-intercept = 0.8 (A1)

(ii) For x-intercepts, $\sin x = -\frac{4}{5} \Rightarrow x = 4.069, 5.356$. (A2)

(iii)



(C2) 5

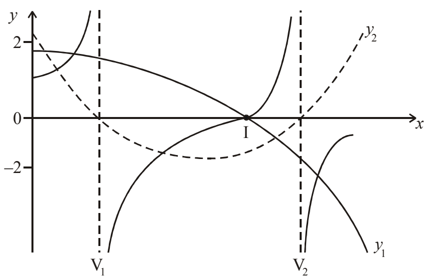
(c) Area = $\int_0^{4.069} \frac{4+5\sin x}{5+4\sin x}\mathrm{d}x - \int_{4.069}^{5.356} \frac{4+5\sin x}{5+4\sin x}\mathrm{d}x$ (M1)(C1)

OR

Area = $\int_0^{5.356} \left|\frac{4+5\sin x}{5+4\sin x}\right|\mathrm{d}x$ (M1)(C1) 2

[17]

5.



(A1)(A1)(A1) (C3)

Note: *Award (A1) for the shape of the graph (all 3 sections), (A1) for both asymptotes (v_1 and v_2), (A1) for the x-intercept I.*

[3]

6. $y = 1 - \frac{8}{x^2 - 5x + 4}$ (M1) (A1)

$= 1 - \frac{8}{(x-4)(x-1)}$ (A1)

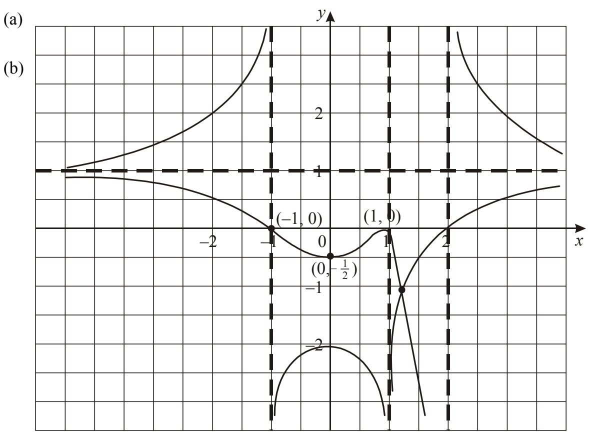
Asymptotes are $y = 1$, (A1) (C2)

$x = 4$, $x = 1$. (A1)(A1)(C2)(C2)

[6]

7. (a) (C3)

(b) (C3)



(A1)(A2)(A3)

Notes: *(a) Award (A1) for the asymptote $x = 2$, (A1) for a correct shape and (A1) for asymptote $y = 1$.*

(b) Award (A1) for each point (–1, 0), $\left(0, -\frac{1}{2}\right)$, (1, 0).

[6]

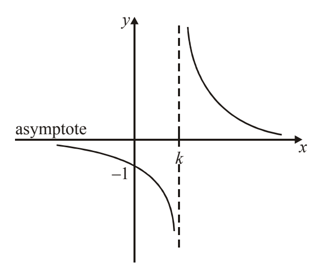
8. (a) $x < -\frac{14}{3}$ $-3 < x < 3$ $x > \frac{14}{3}$ A1A1A1

(b) $-1 < x < -0.800$ or $x > 1$ (accept $-1 < x \le -0.800$) A1A1A1

Note: *Award A1 for the first region, A1 for the second region and A1 for correct inequalities.*

[6]

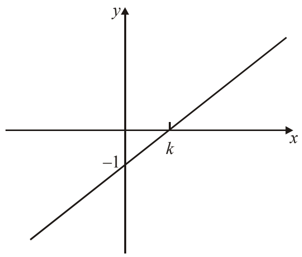
9. (a)



(A1)(A1)(A1)(A1) (C4)

Note: *Award (A1) for the right hand branch, (A1) for the left hand branch passing through (0, –1), (A1) for the vertical* *asymptote through (k, 0) and (A1) for the horizontal asymptote indicated.*

(b)

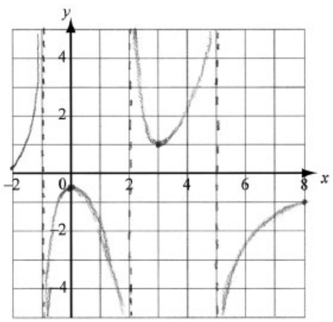


(A1)(A1) (C2)

Notes: *Award (A1) for a straight line with positive gradient through (k, 0), (A1) for value of y-intercept clearly shown. Do not penalize for not showing the discontinuity at x = k.*

[6]

10.



A1A1A1A1A1

Notes: *Award A1 for vertical asymptotes at x = –1, x = 2 and x = 5.*

A1 for $x \to -2,\ \dfrac{1}{f(x)} \to 0^+$

A1 for $x \to 8,\ \dfrac{1}{f(x)} \to -1$

A1 for local maximum at $\left(0, -\dfrac{1}{2}\right)$

(branch containing local max. must be present)

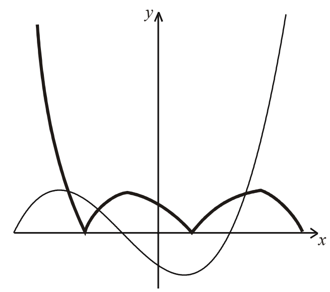
A1 for local minimum at (3, 1) (branch containing local min. must be present)

In each branch, correct asymptotic behaviour must be displayed to obtain the A1.

Disregard any stated horizontal asymptotes such as y = 0 or y = – 1.

[5]

11. (a)



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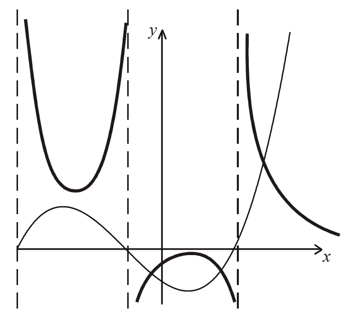
Note: *Award A1 for each branch.*

(b)



A1A1A1

Note: *Award A1 for each branch.*

[6]