IB Vector Problems May 07 – May 08 MARKSCHEME

1. (a) (i) **METHOD 1**

$\vec{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix}$ (A1)

$\vec{AC} = \boldsymbol{c} - \boldsymbol{a} = \begin{pmatrix}3\\0\\1\end{pmatrix} - \begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}2\\-1\\-1\end{pmatrix}$ (A1)

$\vec{AB} \times \vec{AC} = \begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\0 & 1 & 1\\2 & -1 & -1\end{vmatrix}$ M1

$= \boldsymbol{i}\,(-1+1) - \boldsymbol{j}(0-2) + \boldsymbol{k}\,(0-2)$ (A1)

$= 2\boldsymbol{j} - 2\boldsymbol{k}$ A1

Area of triangle ABC $= \frac{1}{2}|2\,\boldsymbol{j} - 2\,\boldsymbol{k}| = \frac{1}{2}\sqrt{8}\left(=\sqrt{2}\right)$ sq. units M1A1

Note: *Allow FT on final A1.*

METHOD 2

$|AB| = \sqrt{2}, |BC| = \sqrt{12}, |AC| = \sqrt{6}$

A1A1A1

Using cosine rule, *eg* on $\hat{C}$ M1

$\cos C = \frac{6+12-2}{2\sqrt{72}} = \frac{2\sqrt{2}}{3}$ A1

$\therefore \text{Area}\,\Delta\text{ABC} = \frac{1}{2}ab\sin C$ M1

$= \frac{1}{2}\sqrt{12}\sqrt{6}\sin\left(\arccos\frac{2\sqrt{2}}{3}\right)$

$= 3\sqrt{2}\sin\left(\arccos\frac{2\sqrt{2}}{3}\right)\ \left(=\sqrt{2}\right)$ A1

Note: *Allow FT on final A1.*

(ii) AB $= \sqrt{2}$ A1

$\sqrt{2} = \frac{1}{2}\text{AB}\times h = \frac{1}{2}\sqrt{2}\times h, h$ equals the shortest distance (M1)

$\Rightarrow h = 2$ A1

(iii) **METHOD 1**

π has form $\boldsymbol{r}\bullet\begin{pmatrix}0\\2\\-2\end{pmatrix} = d$ (M1)

Since (1, 1, 2) is on the plane

$d = \begin{pmatrix}1\\1\\2\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-2\end{pmatrix} = 2-4 = -2$ M1A1

Hence $\boldsymbol{r}\bullet\begin{pmatrix}0\\2\\-2\end{pmatrix} = -2$

$2y - 2z = -2$ (or $y - z = -1$) A1

METHOD 2

$\boldsymbol{r} = \begin{pmatrix}1\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\1\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\-1\end{pmatrix}$ (M1)

$x = 1 + 2\mu$ (i)

$y = 1 + \lambda - \mu$ (ii)

$z = 2 + \lambda - \mu$ (iii) A1

Note: *Award A1 for all three correct, A0 otherwise.*

From (i) $\mu = \frac{x-1}{2}$

substitute in (ii) $y = 1 + \lambda - \left(\frac{x-1}{2}\right)$

$\Rightarrow \lambda = y - 1 + \left(\frac{x-1}{2}\right)$

substitute λ and μ in (iii) M1

$\Rightarrow z = 2 + y - 1 + \left(\frac{x-1}{2}\right) - \left(\frac{x-1}{2}\right)$

$\Rightarrow y - z = -1$ A1

(b) (i) The equation of OD is

$\boldsymbol{r} = \lambda\begin{pmatrix}0\\2\\-2\end{pmatrix}, \left(\text{or } \boldsymbol{r} = \lambda\begin{pmatrix}0\\1\\-1\end{pmatrix}\right)$ M1

This meets π where

$2\lambda + 2\lambda = -1$ (M1)

$\lambda = -\frac{1}{4}$ A1

Coordinates of D are $\left(0, -\frac{1}{2}, \frac{1}{2}\right)$ A1

(ii) $\left|\vec{OD}\right| = \sqrt{0+\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$ (M1)A1

[20]

2. **METHOD 1**

Use of $|\boldsymbol{a}\times\boldsymbol{b}| = |\boldsymbol{a}|\,|\boldsymbol{b}|\sin\theta$ (M1)

$|\boldsymbol{a}\times\boldsymbol{b}|^2 = |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2\sin^2\theta$ (A1)

Note: *Only one of the first two marks can be implied.*

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2(1-\cos^2\theta)$ A1

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2\cos^2\theta$ (A1)

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2 - (|\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta)^2$ (A1)

Note: *Only one of the above two A1 marks can be implied.*

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2$ A1

Hence LHS = RHS AG N0

METHOD 2

Use of $\boldsymbol{a}\bullet\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta$ (M1)

$|\boldsymbol{a}|^2\,|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2 = |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2 - (|\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta)^2$ (A1)

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2\cos^2\theta$ (A1)

Note: *Only one of the above two A1 marks can be implied.*

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2(1-\cos^2\theta)$ A1

$= |\boldsymbol{a}|^2\,|\boldsymbol{b}|^2\sin^2\theta$ A1

$= |\boldsymbol{a}\times\boldsymbol{b}|^2$ A1

Hence LHS = RHS AG N0

Notes: *Candidates who independently correctly simplify both sides and show that LHS = RHS should be awarded full marks.*

If the candidate starts off with expression that they are trying to prove and concludes that $\sin^2\theta = (1-\cos^2\theta)$ award M1A1A1A1A0A0.

If the candidate uses two general 3D vectors and explicitly finds the expressions correctly award full marks. Use of 2D vectors gains a maximum of 2 marks.

If two specific vectors are used no marks are gained.

[6]

3. (a) Use of $\cos\theta = \dfrac{\vec{OA}\bullet\vec{AB}}{\left|\vec{OA}\right|\left|\vec{AB}\right|}$ (M1)

$\vec{AB} = \boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k}$ A1

$\left|\vec{AB}\right| = \sqrt{3}$ **and** $\left|\vec{OA}\right| = 3\sqrt{2}$ A1

$\vec{OA}\bullet\vec{AB} = 6$ A1

substituting gives $\cos\theta = \dfrac{2}{\sqrt{6}}\left(=\dfrac{\sqrt{6}}{3}\right)$ or equivalent M1 N1

(b) L_1: $\boldsymbol{r} = \vec{OA} + s\vec{AB}$ or equivalent (M1)

L_1: $\boldsymbol{r} = \boldsymbol{i}-\boldsymbol{j}+4\boldsymbol{k}+s(\boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k})$ or equivalent A1

Note: *Award (M1)A0 for omitting "**r** =" in the final answer.*

(c) Equating components and forming equations involving s and t (M1)

$1+s=2+2t$, $-1-s=4+t$, $4+s=7+3t$

Having two of the above three equations A1A1

Attempting to solve for s or t (M1)

Finding either $s=-3$ or $t=-2$ A1

Explicitly showing that these values satisfy the third equation R1

Point of intersection is $(-2, 2, 1)$ A1 N1

Note: *Position vector is not acceptable for final A1.*

(d) **METHOD 1**

$\boldsymbol{r} = \begin{pmatrix}1\\-1\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\3\end{pmatrix} + \mu\begin{pmatrix}-3\\3\\-3\end{pmatrix}$ (A1)

$x = 1+2\lambda-3\mu$, $y = -1+\lambda+3\mu$ and $z = 4+3\lambda-3\mu$ M1A1

Elimination of the parameters M1

$x+y = 3\lambda$ so $4(x+y) = 12\lambda$ and $y+z = 4\lambda+3$ so $3(y+z) = 12\lambda+9$

$3(y+z) = 4(x+y)+9$ A1

Cartesian equation of plane is $4x+y-3z=-9$ (or equivalent) A1 N1

METHOD 2

EITHER

The point (2, 4, 7) lies on the plane.

The vector joining (2, 4, 7) and (1, – 1, 4) and $2\boldsymbol{i}+\boldsymbol{j}+3\boldsymbol{k}$ are parallel to the plane. So they are perpendicular to the normal to the plane.

$(\boldsymbol{i}-\boldsymbol{j}+4\boldsymbol{k}) - (2\boldsymbol{i}+4\boldsymbol{j}+7\boldsymbol{k}) = -\boldsymbol{i}-5\boldsymbol{j}-3\boldsymbol{k}$ (A1)

$$\boldsymbol{n}=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ -1 & -5 & -3\\ 2 & 1 & 3\end{vmatrix}$$ M1

$= -12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k}$ or equivalent parallel vector A1

OR

L_1 and L_2 intersect at D (–2, 2,1)

$\overrightarrow{AD} = (-2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) - (\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) = -3\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$ (A1)

$$\boldsymbol{n}=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ 2 & 1 & 3\\ -3 & 3 & -3\end{vmatrix}$$ M1

$= -12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k}$ or equivalent parallel vector A1

THEN

$r \bullet n = (\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) \bullet (-12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k})$ M1

$= 27$ A1

Cartesian equation of plane is $4x + y - 3z = -9$ (or equivalent) A1 N1

[20]

4. The normal vector to the plane is $\begin{pmatrix}1\\3\\2\end{pmatrix}$. (A1)

EITHER

θ is the angle between the line and the normal to the plane.

$$\cos\theta = \frac{\begin{pmatrix}4\\1\\-2\end{pmatrix}\bullet\begin{pmatrix}1\\3\\2\end{pmatrix}}{\sqrt{14}\sqrt{21}} = \frac{3}{\sqrt{14}\sqrt{21}} \quad \left(=\frac{3}{7\sqrt{6}}\right)$$ (M1)A1A1

$\Rightarrow \theta = 79.9°$ (= 1.394 ...) A1

The required angle is 10.1° (= 0.176) A1

OR

ϕ is the angle between the line and the plane.

$$\sin\phi = \frac{\begin{pmatrix}4\\1\\-2\end{pmatrix}\bullet\begin{pmatrix}1\\3\\2\end{pmatrix}}{\sqrt{14}\sqrt{21}} = \frac{3}{\sqrt{14}\sqrt{21}}$$ (M1)A1A1

$\phi = 10.1°$ (= 0.176) A2

[6]

5. **METHOD 1**

(from GDC)

$$\left(\begin{array}{ccc|c}1 & 0 & \frac{1}{6} & -\frac{1}{12}\\ 0 & 1 & -\frac{2}{3} & -\frac{1}{6}\\ 0 & 0 & 0 & 0\end{array}\right)$$ (M1)

$x + \frac{1}{6}\lambda = -\frac{1}{12}$ A1

$y - \frac{2}{3}\lambda = -\frac{1}{6}$ A1

$\boldsymbol{r} = \left(-\frac{1}{12}\boldsymbol{i} - \frac{1}{6}\boldsymbol{j}\right) + \lambda\left(-\frac{1}{6}\boldsymbol{i} + \frac{2}{3}j + \boldsymbol{k}\right)$ A1A1A1 N3

METHOD 2

(Elimination method either for equations or row reduction of matrix)

Eliminating one of the variables M1A1
Finding a point on the line (M1)A1
Finding the direction of the line M1
The **vector** equation of the line A1 N3

[6]

6. Eliminating any one of the variables M1
Using the first two equations this could be $y - 2z = 1$ A1
Let $z = \alpha$ M1
$\Rightarrow y = 2\alpha + 1$ A1

From the first equation $x = 3 + 2\alpha + 1 + \alpha$ M1

$\Rightarrow x = 3\alpha + 4$ A1

Hence $x = 3\alpha + 4$, $y = 2\alpha + 1$, $z = \alpha$

OR $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}4\\1\\0\end{pmatrix} + \alpha\begin{pmatrix}3\\2\\1\end{pmatrix}$

OR $\frac{x-4}{3} = \frac{y-1}{2} = \frac{z}{1}$

OR $x = \frac{3\alpha+5}{2}, y = \alpha, z = \frac{\alpha-1}{2}$

OR $x = \alpha, y = \frac{2\alpha-5}{3}, z = \frac{\alpha-4}{3}$

[6]

7. (a) $l_1\ r=\begin{pmatrix}4+\lambda\\3+5\lambda\\-2\lambda\end{pmatrix}$

for l_1 for $x = 2$, $\lambda = -2$ A1

$\Rightarrow y = -7$

$\Rightarrow z = 4$

Therefore point fits on line. R1

(b) $4 + \lambda = 2$ Eq (1)
$3 + 5\lambda = -1 + 2\mu$ Eq (2)
$-2\lambda = 3 - 3\mu$ Eq (3) (M1)

From Eq (1), $\lambda = -2$ A1

From Eq (2), $3 - 10 = -1 + 2\mu$

$-7 = -1 + 2\mu$

$\mu = -3$ A1

Substituting in Eq (3)

$\Rightarrow 4 = 3 + 9$

$\Rightarrow$ lines do not intersect R1 N0

[6]

8. **EITHER**

The parametric equations of the line are

$x = 4 - \lambda$ A1
$y = -2 + 2\lambda$ A1
$z = 6 + 4\lambda$ A1

Substituting into the left handside of the equation of the plane

$2(4 - \lambda) - (-2 + 2\lambda) + (6 + 4\lambda) = 8 - 2\lambda + 2 - 2\lambda + 6 + 4\lambda$ M1A1

$= 16$

This equals the right handside.
Hence the line is contained in the plane. R1

OR

We first need to prove that that the line and the plane are parallel.
If true, the scalar product is zero.

$\begin{pmatrix}-1\\2\\4\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=-2-2+4=0$ M1A1

Now we need to show that a point on the line lies in the plane.
A point on the line is (4, –2, 6) A1

$\begin{pmatrix}4\\-2\\6\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=8+2+6=16$ M1A1

Hence this is true.
Therefore the line is contained in the plane. R1

9. $\boldsymbol{a}\bullet\boldsymbol{b}=\begin{pmatrix}2\\3\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\5\end{pmatrix}$

$= 3$ (A1)

$\boldsymbol{s}= 3\begin{pmatrix}3\\1\\\lambda\end{pmatrix}+\begin{pmatrix}\mu\\-2\\1\end{pmatrix}$ M1

Note: *Allow FT on* $\boldsymbol{a}\bullet\boldsymbol{b}$ *provided* $\boldsymbol{a}\bullet\boldsymbol{b}$ *is scalar.*

$\boldsymbol{s}=\begin{pmatrix}9+\mu\\1\\3\lambda+1\end{pmatrix}$ A1

$\boldsymbol{s}\bullet\boldsymbol{a}=0\Rightarrow\begin{pmatrix}9+\mu\\1\\3\lambda+1\end{pmatrix}\bullet\begin{pmatrix}2\\3\\-1\end{pmatrix}=0$ (M1)

Note: *Allow FT for* **s**.

$18 + 2\mu + 3 - 3\lambda - 1 = 0\ (20 + 2\mu = 3\lambda)$ A1

$\lambda=\dfrac{20+2\mu}{3}$ A1 N3

[6]

10. $\boldsymbol{n}_1 = -2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{n}_2 = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ (A1)(A1)

$|\boldsymbol{n}_1|=\sqrt{6}$ **and** $|\boldsymbol{n}_2|=\sqrt{6}$ (A1)

$\cos\theta=\dfrac{\boldsymbol{n}_1\bullet\boldsymbol{n}_2}{|\boldsymbol{n}_1||\boldsymbol{n}_2|}$

$\cos\theta=\dfrac{(-2\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k})\bullet(\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k})}{\sqrt{6}\times\sqrt{6}}$ M1(A1)

$\cos\theta=\dfrac{1}{6}\ (0.167\text{ to }3\text{ sf})$ A1 N4

[6]

11. **METHOD 1**

$\overrightarrow{AB}=2\boldsymbol{j}-\boldsymbol{k}$ and $\overrightarrow{AC}=-3\boldsymbol{i}+2\boldsymbol{j}$ (A1)(A1)

$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\0&2&-1\\-3&2&0\end{vmatrix}$ M1

$= 2\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$ A1

Area Δ ABC = $\frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|$ (M1)

= $\frac{7}{2}$ A1 N4

METHOD 2

$\vec{AB}=2\boldsymbol{j}-\boldsymbol{k}$ and $\vec{AC}=-3\boldsymbol{i}+2\boldsymbol{j}$ (A1)(A1)

Attempting to use the scalar product to find θ *ie*
$\vec{AB}\bullet\vec{AC}=\left|\vec{AB}\right|\left|\vec{AC}\right|\cos\theta$ M1

$\cos\theta=\frac{4}{\sqrt{65}}$ $\left(\theta=\arccos\frac{4}{\sqrt{65}}=\arcsin\frac{7}{\sqrt{65}}=60.255...°\right)$ A1

Area Δ ABC = $\frac{1}{2}\left|\vec{AB}\right|\left|\vec{AC}\right|\sin\theta$ $\left(=\frac{1}{2}\times\sqrt{5}\times\sqrt{13}\times\frac{7}{\sqrt{65}}\right)$ (M1)

= $\frac{7}{2}$ A1 N4

[6]

12. (a) (i) $\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ -2 & 2 & 3\\ 2 & 1 & 0\end{vmatrix}=-3\boldsymbol{i}+6\boldsymbol{j}-6\boldsymbol{k}$ M1A1 N2

(ii) $\boldsymbol{n}=\boldsymbol{i}-2\boldsymbol{j}+2\boldsymbol{k}$ so equation of π_1 is
$x-2y+2z=D$ (or $\boldsymbol{r}\bullet\boldsymbol{n}=\boldsymbol{a}\bullet\boldsymbol{n}$) M1
substituting (3, 1, 5) $\Rightarrow 3-2+10=D$ M1
so $11=D$
Therefore the equation of plane π_1 is $x-2y+2z=11$ AG N0

(b) Using scalar product (M1)
of normal vectors (M1)

$\begin{pmatrix}1\\-2\\2\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\-2\end{pmatrix}=2+2-4=0$ A1

The normals are perpendicular, so the planes are perpendicular. R1 AG

(c) **METHOD 1**
Elimination of one variable. (M1)
Choosing a parameter. (M1)

METHOD 2
Finding direction of the line (M1)
Finding a point on the line (M1)

$x=2\lambda-1$; $y=2\lambda-6$; $z=\lambda$ A1A1A1 N5

OR $\left(x=\mu+5, y=\mu, z=\frac{1}{2}\mu+3\right)$

OR $\left(x=t, y=t-5, z=\frac{1}{2}t+\frac{1}{2}\right)$

OR $\left(\boldsymbol{r}=\begin{pmatrix}-1\\-6\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\1\end{pmatrix}\right)$

(d) Direction vector of $l_2=2\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}$ (A1)

equation of $l_2:\frac{x-3}{2}=\frac{y+5}{2}=\frac{z+1}{1}$ A1A1 N3

(e) (i) Recognizing that the equation of line (PQ) is needed (M1)

equation of line (PQ) is $\boldsymbol{r}=\begin{pmatrix}3\\-5\\-1\end{pmatrix}+t\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ (A1)

Q on $\pi_2 \Rightarrow 2(3+2t)-(-5-t)-2(-1-2t)=4$ M1
$t=-1$ A1
$\Rightarrow Q=(1, -4, 1)$ A1 N0

(ii) PQ = $\sqrt{(3-1)^2+(-5+4)^2+(-1-1)^2}$ (M1)
= 3 A1 N2

[23]

13. (a) Substituting for $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ into $\boldsymbol{c}=m\boldsymbol{a}+n\boldsymbol{b}$ (M1)
Forming any 2 of the following equations A1A1
$m+n=2$ Eq(1)
$-m+2n=-5$ Eq(2)
$m+4n=-1$ Eq(3)

Note: *Accept equations in vector form.*

Solving for m and n (M1)
$m=3$ and $n=-1$ A1 N3

(b) **METHOD 1**

$\boldsymbol{a}\times\boldsymbol{b}=\begin{vmatrix}\mathrm{i} & \mathrm{j} & \mathrm{k}\\ 1 & -1 & 1\\ 1 & 2 & 4\end{vmatrix}$ (M1)(A1)

$=-6\boldsymbol{i}-3\boldsymbol{j}+3\boldsymbol{k}$ A1

Attempting to find $|\boldsymbol{a}\times\boldsymbol{b}|$ $\left(=\sqrt{54}=3\sqrt{6}\right)$ (M1)

$u = \frac{1}{\sqrt{54}}(-6i - 3j + 3k)$ $\left(= \frac{1}{\sqrt{6}}(-2i - j + k)\right)$ A1 N3

Note: *Award as above for* $b \times a = 6i + 3j - 3k$ *and* $u = \frac{1}{\sqrt{54}}(-6i - 3j + 3k)$.

METHOD 2

Stating 2 equations derived from $a \bullet u$ and $b \bullet u$ where $u = xi + yj + zk$. A1

$x - y + z = 0$ Eq(1)

$x + 2y + 4z = 0$ Eq(2)

Attempting to solve the above system of equations (M1)

Solution sets include

$x = -2z$ and $y = -z$ A1

OR $y = \frac{x}{2}$ and $z = \frac{-x}{2}$ A1

OR $z = -y$ and $x = 2y$ A1

Note: *Accept any correct numerical solution such as* $x = 2, y = 1, z = -1$.

Using $x^2 + y^2 + z^2 = 1$ (ie $|u| = 1$) to find values for x, y and z. (M1)

Either $u = \frac{1}{\sqrt{6}}(2i + j - k)$ or $u = \frac{1}{\sqrt{6}}(-2i - j + k)$ A1 N3

Note: *Ignore any additional answers, even if incorrect.*

(c) (i) **METHOD 1**

Equation of π_1 is of the form $x + 2y + 4z = d$ (M1)

Substituting $(1, -1, 1)$ ($\Rightarrow d = 3$) M1

$\Rightarrow$ ($x + 2y + 4z = 3$ A1 N3

METHOD 2

$r \bullet (i + 2j + 4k) = d$ (M1)

Evaluate the scalar product $a \bullet (i + 2j + 4k)$ (= 3) M1

$\Rightarrow x + 2y + 4z = 3$ A1 N3

(ii) L(3, 0, 0), M$\left(0, \frac{3}{2}, 0\right)$ and N $\left(0, 0, \frac{3}{4}\right)$ (M1)A1

(d) (i) P has coordinates $(x, y, z) = (\lambda, 2\lambda, 4\lambda)$ (A1)

Substituting the coordinates of P into the equation of π_1 (M1)

$\lambda + 4\lambda + 16\lambda = 3$ A1

$\lambda = \frac{1}{7}$ (A1)

$P\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$ A1 N3

(ii) Distance = $\frac{1}{7}\sqrt{1^2 + 2^2 + 4^2}$ M1

$= \frac{\sqrt{21}}{7}$ or equivalent (= 0.655) A1 N2

Note: *Award M0A0 for any other method.*

(e) (Given θ is the angle between π_2 and a line and α is the angle between the normal and a line) $\cos\alpha = \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ (R1)

Using the scalar product *eg* $\sin\theta = \frac{a \bullet b}{|a||b|}$ or $\cos\alpha = \frac{a \bullet b}{|a||b|}$ M1

$(\sin\theta) = \frac{(i - j + k)\bullet(i + 2j + 4k)}{|i - j + k||i + 2j + 4k|}$ (A1)

$= \frac{1}{\sqrt{7}}$ (or equivalent) (A1)

$\theta = 0.388$ (= 22.2°) $\left(= \arcsin\frac{1}{\sqrt{7}}\right)$ A1 N2

[27]